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Evaluation of the Munich Method for Modeling Rocket Engine Performance

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A new procedure, dubbed the Munich Method, has been proposed recently for the modeling of rocket engine performance. The author of the Munich Method claims it to be an extension and improvement of the thermodynamic procedures used to model rocket engines in the NASA-Lewis chemical equilibrium program. An examination of the Munich Method shows that it contains several flaws. If these defects are corrected then the Munich Method will produce results identical to those generated by the NASA-Lewis Code.

Nomenclature

A = cross-sectional area of a rocket engine
$a_{\alpha i}$ = number of atoms of element α in species i
b_{α}^{0} = gram atoms of element α per unit mass
C_p = molar heat capacity at constant pressure and composition
c_p = specific heat at constant pressure and composition
H_i = partial molar enthalpy of the ith species
h = specific enthalpy
h_F = injection specific enthalpy
L = number of chemical elements
M = mean molecular weight
M_i = molecular weight of the ith species
m = mass flow rate
N = number of chemical species
n_i = moles of species i per unit mass
p = pressure
p_F = injection pressure
R = universal gas constant
S_i = partial molar entropy of the ith species
s = specific entropy
T = thermodynamic temperature
u = flow velocity
v = specific volume
w_i = weight fraction of the ith species
x_i = mole fraction of the ith species
μ_i = chemical potential of the ith species, $H_i - TS_i$
π_α = Lagrangian multiplier for element α
π_E = Lagrangian multiplier for energy
π_M = Lagrangian multiplier for momentum

Introduction

THE NASA-Lewis Code for the thermodynamic calculation of reacting system properties has a long history dating from the very first version in 1959 to its most recent revision in 1988.[1–7] In that time-span it has achieved worldwide distribution and has come to be regarded, by many, as the program of choice for performing those thermodynamic calculations which it offers. The available options are 1) equilibrium compositions, and the one-dimensional thermodynamic modeling of 2) gaseous detonations, 3) shock waves, and 4) rocket engine performance. The program uses standard thermodynamics and standard one-dimensional flow equations, and has never elicited any controversy. In fact, any issues of consequence were throught to have been resolved a quarter of a century ago. However, a book[8] has been published recently which claims to be a fundamental look at the procedures used in the NASA-Lewis Code to model the performance of chemical rocket engines. In particular, the express purpose of that book (p. 7) is to present an "analysis and correction of" the NASA-Lewis Code for the thermodynamic modeling of rocket performance. The Munich Method is the soubriquet the author of the book confers on his procedure.

Throughout his book the author of the Munich Method makes some strong, unsupported allegations about the deficiencies of the NASA-Lewis computer program. For example, he cannot understand (p. 57) why the code is not "officially questioned because of its serious theoretical errors." He asserts (p. 67) that the program is based on a theory "whose premises are no longer known, or cannot be assessed, or are purposely ignored." Moreover, he avers (p. 95) that there occur "in practical applications of the NASA-Lewis Code, unexpected ambiguities in the results." Incredibly, there is even the intimation (p. 12) that the Lewis Code is responsible for the Challenger disaster. The author offers the Munich Method as a replacement for the rocket performance calculations of the Lewis Code. Those who might be skeptical of the Munich Method as a substitute for the Lewis Code are admonished by the author (p. 100) that "At least they are informed and will be spared a retreat to the status quo ante."

At the technical level, the author contends that the Lewis Code ignores the pressure drop in a finite-area combustion chamber, and also that it does not require the constancy of the steady mass flow rate in the rocket nozzle (p. 48). He also objects to the use of Gibbs energy minimization in the Lewis Code to determine equilibrium states in the nozzle. It seems the author is troubled that the temperature and pressure are changing during the flow through a rocket, and concludes that Gibbs energy minimization is (p. 92) "theoretically inadmissible" and is (p. 158) an "untenable" and "theoretically crude approximation." Moreover, he charges (p. 250) that the program's thermodynamic data "have not been checked . . . in over twenty years, let alone altered."

Some of his misperceptions must simply reflect the fact that he is probably working with a copy of the program that has been superseded by several more recent versions. Clearly, the available references[1–7] and the many current users of the program attest to the fact that the program does account for finite-area combustion chambers, and that both the program and its data have been regularly updated. Other faulty perceptions must stem from misconstruing the program documentation. For example, he finds fault with the procedure

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used to calculate the nozzle properties as a function of area ratio (p. 97). This occurs in spite of the fact that the method routinely converges to better than 0.004% in the pressure ratio at each point of the nozzle. One possible explanation for this is that he perhaps fails to make a distinction between initial estimates and the converged results.

A review[9] of the book has been published recently. That review discussed the book only in general terms and did not address the technical aspects of the Munich Method. However, among other things, it concluded that "The book is difficult to read or digest, and the explanations for the basic phenomena are sometimes not very lucid. Some statements made about the Navier-Stokes equations are questionable." It also said that "The book is strictly for experts in thermo-fluiddynamics. Even for them it will probably require several weeks of intensive study . . . (to) truly comprehend the book."

The purpose of this paper is to examine the technical aspects of the Munich Method and its claim to be an improvement over the NASA-Lewis Code. There are three relevant questions about the Munich Method which must be addressed.

1) Are the governing equations thermodynamically and mathematically correct?

2) Are the working equations, whether correct or incorrect, implemented in a numerically correct manner?

3) Regardless of the answers to questions 1) and 2), are the predictions of the proposed method an improvement over those given by the Lewis Code?

Alternative Theory and the Munich Method

Much of the book[8] is irrelevant to its avowed purpose. It is poorly organized, and suffers from a poor translation from the original German. To place its charges into some context, it is necessary to review its contents briefly. The book is divided into two parts. A substantial portion of the first part of the book, and all of Appendix 1, are devoted to the trivial case of the one-dimensional flow of a nonreacting, "relaxing model gas." Apparently the intent is to demonstrate that the heat capacity ratio and the isentropic exponent are unequal when there is a "conversion process." This has been known to some people for at least 40 yr and, perhaps, longer.

The second portion of the book is dedicated to what is called the Alternative Theory and to the Munich Method for doing rocket performance calculations. According to the author (p. 121), the Alternative Theory "is a phenomenological continuum theory of real compressible fluids." Furthermore, (p. 133) "fundamental elements of the Alternative Theory serve as the basis for the set of premises used for the Munich Method." Apparently such a theory is necessary, in the author's view, since he erroneously contends (p. 124) that the Navier-Stokes equations are "valid only for an incompressible model fluid." Actually, the Alternative Theory is nothing more than a collection of unsupported statements which culminate in an equation which is named the Navier-Saint Venant equation. This appellation is conferred on the equation despite the fact that neither Navier nor Saint Venant had anything to do with it. The equation, which is not derived in the book, is never applied to modeling rocket engine performance or anything else.

The Munich Method, when reduced to its essentials, is simply the prescription that all equilibrium states in a rocket engine should be determined by maximizing the entropy rather than minimizing the Gibbs energy. Apparently, the author is unaware that the two procedures produce identical results when properly executed. These are only two of several alternative, but equivalent, formulations of thermodynamics as is made clear in a number of thermodynamic texts. Nevertheless, the author declares that the combustion state must be determined by maximizing the entropy so as to satisfy the one-dimensional energy and momentum equations for a prescribed mass flow rate and a specified cross-sectional area for the combustion chamber (p. 139ff). When it comes to calculating the converging-diverging portions of the rocket engine, he contends that the calculation must be carried out with the same mass flow rate as was used in the combustion calculation. He refers (p. 48) to this constancy of the mass flow rate as an "eigenvalue" rather than simply a constant parameter for the calculation. His prescription is to integrate what he refers to as the "nozzle differential equation" (p. 147, 161) which is nothing more than the partial derivative of the pressure with respect to temperature at constant entropy. Compositions along the converging-diverging portions of the rocket engine are to be calculated by a minimization of the Gibbs energy at the local temperature and pressure obtained by an integration of the "nozzle differential equation." The rationale for using the Gibbs energy here, even though it contravenes his entropy maximization dictum, is that it provides a "substitute process for the linkage between two states (p. 158)." The mass flow rate is determined iteratively by doing successive combustion and throat calculations until the velocity at the throat converges to the sonic velocity (p. 160). A similar iterative computation between the throat and the nozzle exit determines the exit properties so as to maintain the mass flow rate fixed at the throat value while conserving energy (p. 164).

This brief outline of the Munich Method shows it to be pretty much standard thermodynamic fare, independent of any alternative theory and, if implemented correctly, cannot yield results which differ from those produced by the NASA-Lewis Code. As such, the method is incapable of supplying any new insight into rocket engine design. Therefore, the issue devolves to the evaluation of the correctness and efficiency of implementation. The following sections of this paper will examine the combustion equations, the nozzle calculations, and the numerics in greater detail.

Combustion Equations for the Munich Method

According to the Munich Method, the combustion state is to be determined by maximizing the entropy, regarded as a function of temperature, pressure, and composition. The maximum is subject to the constraints imposed by the one-dimensional energy and momentum equations, as well as to the constraint of elemental conservation. The entropy maximum criterion for equilibrium is normally written by considering the entropy to be a function of its natural variables which, for a fluid, are internal energy, volume, and composition. However, it can be demonstrated that the entropy maximum principle is valid when temperature and pressure replace the internal energy and volume as independent variables. This is simply a consequence of the behavior of an extremum of a function under a change in independent variables.

The equations of the Munich Method for rocket engine combustion (p. 139, 140) actually come from the necessary conditions for the constrained minimum of the negative entropy using the method of undetermined Lagrangian multipliers. There are $L + 2$ constraints for a system composed of L chemical elements and these constraints impose the conservation of momentum, energy, and the elements.

Momentum

$$p + (m/A)^2 v = p_F$$

Energy

$$h + (m/A)^2 v^2/2 = h_F$$

Element Conservation

$$\sum_i a_{\alpha i} n_i = b_\alpha^0 \qquad \alpha = 1, 2, \ldots, L$$

The first two of these constraints contain the mass flow rate per unit area which is a parameter of the problem. In Ref. 8, p_F and h_F are referred to as the fluid state pressure and enthalpy. More commonly, they would be called the injection pressure and injection enthalpy, and correspond to the pressure and enthalpy in the combustion chamber at the face of the injector. The Lagrangian function for the minimization is formed as a linear combination of the negative entropy and the constraints:

$$\Phi = -s(T, p, n_j) + \sum_{\alpha} \pi_{\alpha} \left(\sum_{j} a_{\alpha j} n_j - b_{\alpha}^{0} \right)$$
$$+ \pi_E[h + (m/A)^2 v^2/2 - h_F]$$
$$+ \pi_M[p + (m/A)^2 v - p_F]$$

This Lagrangian is shown as Eq. (3.17) on p. 139. It also appears on p. 88 except that there it appears with an additional term which is irrelevant for the present discussion. The choice of sign for the multipliers associated with the elemental constraints is the same in both places and differs from the choice made here. The reason for the different choice of sign here will become apparent later. To complete the list of equations, the Munich Method uses the ideal gas equation of state:

$$v = RT/pM$$

The mean molecular weight is expressible in terms of the species molecular weights and the composition variables:

$$M = \sum_{j} M_j n_j \Big/ \sum_{k} n_k$$

Since the composition variables are defined to be per unit mass, the numerator in this expression is equal to unity. However, I shall not use this fact in generating the necessary conditions for an extremum.

The necessary conditions for the extremum are obtained simply by differentiating the Lagrangian function with respect to the composition, temperature, pressure, and the multipliers, and then setting the partial derivatives to zero. The resulting system of equations for the extremum have the following form for a system composed of N species:

$$\partial\Phi/\partial n_i = 0 = -S_i + \sum_{\alpha} \pi_{\alpha} a_{\alpha i}$$
$$+ \pi_E \left[H_i + (m/A)^2 v^2 (1 - M_i/M) \Big/ \sum_{k} n_k \right]$$
$$+ \pi_M \left[(m/A)^2 v (1 - M_i/M) \Big/ \sum_{k} n_k \right]$$
$$= \mu_i/T + \sum_{\alpha} \pi_{\alpha} a_{\alpha i} + (T\pi_E - 1) H_i/T$$
$$+ (v\pi_E + \pi_M)(m/A)^2 v (1 - M_i/M) \Big/ \sum_{k} n_k$$
$$i = 1, 2, \ldots, N \tag{1}$$

$$\partial\phi/\partial \ln T = 0 = -c_p + \pi_E[Tc_p + (m/A)^2 v^2]$$
$$+ \pi_M (m/A)^2 v = (T\pi_E - 1)c_p + (v\pi_E + \pi_M)(m/A)^2 v \tag{2}$$

$$\partial\Phi/\partial \ln p = 0 = R/M - \pi_E (m/A)^2 v^2$$
$$+ \pi_M[p - (m/A)^2 v]$$
$$= R/M - (v\pi_E + \pi_M)(m/A)^2 v + p\pi_M \tag{3}$$

$$\partial\Phi/\partial\pi_{\alpha} = 0 = \sum_{i} a_{\alpha i} n_i - b_{\alpha}^{0} \qquad \alpha = 1, 2, \ldots, L \tag{4}$$

$$\partial\Phi/\partial\pi_E = 0 = h + (m/A)^2 v^2/2 - h_F \tag{5}$$

$$\partial\Phi/\partial\pi_M = 0 = p + (m/A)^2 v - p_F \tag{6}$$

The Munich Method's version of the necessary conditions are shown explicitly only on page 88 of Ref. 8 and contain contributions from the additional, irrelevant term in the Lagrangian. If these extra terms are ignored, then those equations may be compared to the set of $N + L + 4$ equations shown here. It is immediately apparent that the two sets of equations are almost identical. As a minor point, it should be noted that while both pages 88 and 139 of Ref. 8 indicate that derivatives are calculated with respect to temperature and pressure, the consonance with the equations shown here demonstrates that the derivatives are really with respect to the logarithms of these variables. Apart from the intentional difference in the choice of sign for the Lagrangian multipliers of the elemental constraints, the only real differences between the equations shown here and those on page 88 occur in the derivative of the Lagrangian function with respect to composition. These variances are caused by errors in the Munich Method equation shown on page 88. Thus, partial specific entropy and enthalpy appear there in place of the partial molar quantities shown here. Partial specific quantities cannot be correct. These would render the equation dimensionally inconsistent, since it was generated by differentiation with respect to moles. Differentiation with respect to moles in the Munich Method is easily confirmed by examining the term arising from the constraint on the elements. A more serious error in the composition derivative involves the derivative of the specific volume with respect to composition. For the ideal gas equation of state, the specific volume depends on composition only through the mean molecular weight. In place of the correct expression

$$(1 - M_i/M) \Big/ \sum_{k} n_k = -\partial \ln M/\partial n_i = \partial \ln v/\partial n_i$$

The corresponding Munich Method equation on page 88 shows the following incorrect quantity:

$$-M_i(1 - x_i) = -M_i \left(1 - n_i \Big/ \sum_{k} n_k \right)$$

While I have uncovered some errors in the combustion equations for the Munich Method, they are not the working equations for that method. Those are shown[8] as Eqs. (3.18) and (3.19) on page 140. It is certainly possible that the discovered errors are merely typographical and that the working equations are correctly deduced from the necessary conditions for an extremum in the entropy. Consequently, I shall temporarily ignore these errors, assume that the equations given here are correct, and attempt to obtain the working equations from them. The author of the Munich Method shows them only for the simplest propellant combination, the cryogenic hydrogen-oxygen system, and some of his algebraic manipulations are predicated on the specific character of that system. Since the general case is no more difficult than the special case, I shall avoid system specific assumptions.

Equation (1) can be replaced by an equation obtained by eliminating the combination $v\pi_E + \pi_M$ between Eqs. (1) and (2), multiplying the result by T, and replacing the specific heat with the molar heat capacity at constant pressure and composition. The two heat capacities have a simple relationship

$$C_p = c_p \Big/ \sum_{k} n_k$$

These manipulations produce an equation which does not contain the Lagrangian multiplier for momentum:

$$\mu_i + T\sum_\alpha \pi_\alpha a_{\alpha i} + (T\pi_E - 1)[H_i + TC_p(M_i - M)/M] = 0 \qquad (7)$$

Similarly, we can replace either Eq. (2) or (3), but not both, with their sum:

$$(T\pi_E - 1)c_p + R/M + p\pi_M = 0 \qquad (8)$$

The elemental constraint equations [Eq.(4)] serve to scale the values of the moles n_i. If one is only interested in relative composition values then it is possible to use one of them, say the last one, to express the conservation of elements in relative form. In place of Eq. (4) we could use the L-1 equations

$$\sum_i [a_{\alpha i} - (b^0_\alpha/b^0_L)a_{Li}]n_i \Big/ \sum_k n_k = 0$$

$$\alpha = 1, 2, \ldots, L - 1 \qquad (9)$$

which are a direct consequence of Eq. (4). However, since we have effectively discarded the last member of Eq. (4) it must be replaced by another equation involving the composition variables. The added equation is simply the identity satisfied by the mole fractions. Mole fractions are easily expressible in terms of the molar composition variables:

$$x_i = n_i \Big/ \sum_k n_k$$

These new composition variables satisfy a very simple equation by virtue of their definition:

$$\sum_k x_k - 1 = 0 \qquad (10)$$

Equations (5–10) are a direct consequence of the Eqs. (1–6) which are themselves the necessary conditions for the existence of a constrained extremum in entropy. However, the set of Eqs. (1–6) has one more equation than the set of Eqs. (5–10). As was pointed out previously, Eqs. (2) and (3) are represented by the single Eq. (8) which is their sum. The missing equation can be thought of as either Eq. (2) or (3), their difference, or some other independent combination of Eqs. (2) and (3). In effect, either Eq. (2) or (3) has been arbitrarily discarded in generating the set of Eqs. (5–10). I shall return to this point shortly, but first let us compare the equations derived here with the working equations of the Munich Method which appear as Eqs. (3.18) and (3.19) on page 140.

If we specialize to the case $L = 2$, then each equation in the set of Eqs. (5–10) corresponds to one member of the working equations of the Munich Method with the exception that the last member of Eq. (3.18) has no counterpart. Corresponding equations are exactly the same except for Eq. (7) and its counterpart. To compare Eq. (7) to its mate we must first relate partial molar properties to partial specific properties. The weight fractions are related to the moles by the simple expression.

$$w_i = M_i n_i$$

and therefore, the ith specific property can be converted to the molar property by multiplication with the species molecular weight. With this information we conclude that Eq. (7) matches its counterpart in the Munich Method except for the molecular weight terms. In the equation for the Munich Method we find the term

$$M_i(1 - x_i)/M$$

in place of the correct combination

$$(M_i - M)/M = \left[M_i(1 - x_i) - \sum_{j \neq i} M_j x_j\right] \Big/ M$$

shown in Eq. (7). This is consistent with the error already observed while discussing the necessary conditions for an extremum. However, what is not in keeping with the earlier discussion is that now the signs on the Lagrangian multipliers π_α agree, whereas, at the start of the derivation they were of opposite sign. While indicative of poor algebra or inadequate proofreading, this is unlikely to be a serious problem for the Munich Method. This is because the Munich Method uses Lagrangian multipliers only to effect a solution of the extremum problem. Derivatives of the Lagrangian multipliers can also be used to generate derivative properties such as the heat capacity and the isentropic exponent for the equilibrium state. These properties are used to calculate nozzle conditions and incorrect values could adversely affect the prediction of nozzle behavior. It seems, however, that the Munich Method's procedures for calculating derivatives are framed in terms of the extent of reaction variables rather than the Lagrangian multipliers (see p. 144ff). Consequently, the erroneous multipliers are probably not an important factor in any results calculated by the Munich Method.

We now return to the saga of the abandoned equation. There is one equation in the set of working equations for the Munich Method which does not correspond to any member of the set [Eqs. (5–10)] and which cannot be a consequence of the necessary conditions for an extremum [Eqs. (1–6)]. It is the last member of Eq. (3.18) and it specifies an entropy value for the working fluid at the exit of the combustion chamber. Thus, it follows that the Munich Method has quite arbitrarily, and unacceptably, discarded one of the necessary conditions for an extremum. As has already been mentioned, the discarded equation can be thought of as either Eq. (2) or (3) or some combination of them. Without ever mentioning that an equation has been discarded, the author of the Munich Method elects to replace the discarded equation with an equation defining the value of the entropy at the exit of the combustion chamber. In particular, the Munich Method chooses the entropy value to be the same as the entropy at the entrance to the combustion chamber. But this then creates two problems of its own. First, it is tantamount to making the claim that the combustion process is isentropic, which is manifestly not the case. Second, it raises the unpleasant specter of having to generate a value for the entropy at the entrance to the combustion chamber. The author of the Munich Method asserts (p. 171, 246) that the correct value corresponds to the entropy of the unreacted fluid mixture, presumably a solution of cryogenic hydrogen and oxygen, and would therefore include the contribution of mixing to the entropy. Clearly, such thermodynamic information is not readily available for this system. If it were, he would quickly learn that the entropy change associated with a mixing process, even in very nonideal solutions, is always much smaller than the entropy generated in combustion. Consequently, the assigned entropy value would be far too small and would produce a combustion state drastically different from what occurs in nature. But, ever resourceful, he proceeds to solve the dilemma of the unknown entropy value in a creative way. He claims (p. 191) that only for the sake of "a fair comparison of the Munich Method with the NASA-Lewis Code," does he deign to approximate the unknown value with the entropy for an adiabatic flame combustion. Naturally he realizes that this "is a (bad) approximation" to what he considers to be the correct value. Of course, it does not hurt that his choice of a "(bad) approximation" guarantees that his calculated results, while still incorrect, are not totally ridiculous. Even if one were prepared to accept the working equations of the Munich Method, its erroneous equations, and its isentropic combustion process, one could question whether the mixture entropy is the appropriate value on purely physical grounds. In the cryogenic hydrogen-oxygen rocket system, the fuel and oxidizer are never mixed until they are injected into the combustion chamber. Hence, the reactants enter the rocket as pure hydrogen and pure oxygen rather than some solution of the two. As a result, the mixing takes place within the combustion chamber and must be thought of as a part of the overall combustion process, which already includes the propellant vaporization process. Consequently, the appropriate reactant properties should then be a composition average of the liquid hydrogen and liquid oxygen properties, rather than the properties of a solution of the two components. This entropy is even lower than that of a solution of the two components under comparable conditions.

I shall conclude this discussion of the combustion equations for the Munich Method by returning to the necessary conditions for an extremum. Equations (2) and (3) can be viewed as a pair of linear equations in the Lagrangian multipliers for energy and momentum, π_E and π_M. The exact solution of these equations is given by

$$\pi_E = 1/T; \qquad \pi_M = -v/T$$

and this may be easily verified by substituting these values into Eqs. (2) and (3). The author of the Munich Method refers to this value of the Lagrangian multiplier for energy as the trivial value (p. 141), and claims to calculate numerically a value for the numerator which is about 1% smaller, namely, about $0.99/T$. This, of course, was only done for the hydrogen-oxygen system. Clearly, the Munich Method is in error since the pair of equations is of full rank and the solution given here is the only solution. However, there is an even more interesting conclusion which follows from the values of these Lagrangian multipliers. If they are substituted into Eq. (1), then only the first two terms in the second equality survive. These equations may now be divided by the universal gas constant

$$\mu_i/RT + \sum_{\alpha} \lambda_\alpha a_{\alpha i} = 0 \qquad \lambda_\alpha \equiv \pi_\alpha/RT \tag{11}$$

Equations (11), when coupled with the element constraints Eq. (4), are exactly those which have always been used in the Lewis Code to determine equilibrium compositions. Furthermore, to this set of equations add the remaining constraint equations, Eqs. (5) and (6), and we have precisely those equations which are used in the latest version of the Lewis Code to calculate ideal rocket performance for finite-area combustors. This establishes analytically the assertion made earlier that a correct implementation of the Munich Method must agree exactly with the Lewis Code.

Nozzle Calculations for the Munich Method

The Munich Method for calculating the converging-diverging portions of a rocket engine requires the integration of what is referred to as the "nozzle differential equation" (Eq. 3.39, p. 147 and Eq. 3.56, p. 161). This equation is merely the partial derivative of the logarithm of the pressure with respect to the logarithm of the temperature at constant entropy. Its sole purpose is to define pressure as a function of temperature along the nozzle, for an isentropic process, starting at some initial position and extending to some terminal position. For example, the initial position could be the combustion state, while, the terminal position might be the nozzle throat. The desired terminal state is defined by the nozzle cross-sectional area, the constancy of mass flow rate, and the requirement for energy conservation. The satisfaction of the energy equation is used as the criterion for termination of the integration process. If the terminal state is the throat, then the flow velocity is compared to the sonic velocity. Any discrepancy requires an adjustment of the mass flow rate and a repeat of the calculation of combustion and throat until convergence.

The procedure is correct in principle but execrable in practice. It is excessively, and unnecessarily, expensive because the integration requires the determination of the local equilibrium state at each point along the path of integration. The author himself observes (p. 168) that the relations are "strongly non-linear and require relatively considerable computing time." The procedure is also prone to numerical error because integrating a differential equation is more difficult than simply solving an algebraic equation. This inaccuracy is clearly evident in the calculated results for the Munich Method which display a nonconstancy of entropy for what is ostensibly an isentropic expansion process. The author observes (p. 210) that "The more numerous the required incremental steps for solving the nozzle differential equation between two states are, the greater the resulting deviations." Yet he does not draw the obvious inference: numerical garbage in the integration. By some inscrutable logic he declares that the "origin of the discrepancies . . . is obvious." They are a consequence of the poor thermodynamic data which he borrowed from the Lewis Code (p. 210). Never mind that the Lewis Code has always maintained a constant entropy during isentropic processes when using its own data base. The author does assure his readers that he recognizes the inherent deficiencies of the Lewis thermodynamic data (p. 168, 210). However, attempts to work with more accurate data had to be abandoned because (p. 168) "the radical increase in computing time made the idea impractical."

Numerics

I shall now turn to an examination of the numerical results calculated by the Munich Method even though, by now, it should be evident that the method, as originally presented and implemented, has some serious theoretical and practical problems. The theoretical problems are caused by grievous errors in thermodynamics and mathematics. These errors have been corrected in the previous sections of this paper and, hence, one could model rocket engines using the corrected equations if one were so inclined. However, there is nothing to be gained since the results would then be identical to those calculated with the NASA-Lewis Code. The practical problems are linked to the method's severe computational inefficiency in modeling the rocket nozzle. It substitutes the integration of an ordinary differential equation, coupled with multiple solutions of the equilibrium equations, for what is normally just the solution of a system of nonlinear equations. My examination of the method's numerics will be limited to the calculations which appear as a part of chapter four in the second part of the book. This chapter is devoted to a comparison of rocket calculations which the author performed both with the Munich Method and with the Lewis Code, apparently using a version which was supplanted by many later releases.

The original combustion equations of the Munich Method have already been demonstrated to be thermodynamically and algebraically incorrect. We now inquire about the correctness of the numerical implementation of these incorrect equations. The Munich Method claims to satisfy both the one-dimensional energy and momentum equations in the combustion state. It is simple to test that assertion using values presented in computer output listings 2–7 (termed "protocols" in the book), which correspond to various rocket engines using liquid hydrogen and liquid oxygen as propellants. The test of the energy equation discloses that the combustion properties as computed by the Munich Method do indeed satisfy the energy equation. A similar test for the momentum equation is shown in Table 1.

From this table it is clear that the combustion properties from the Munich Method do not satisfy the momentum equation. If that were the case, the numbers in columns 3 and 4 of Table 1 would be nearly equal. One could speculate about

Table 1 Munich Method combustion calculations[8]

Listing	$1/v$, Kgm/m^3	u^2/v, MPa	$p_F - p$, MPa	$2(p_F - p)$, MPa
2	6.2882	0.4378	0.2197	0.4394
3	1.5588	0.1872	0.0949	0.1898
4	3.7656	0.7191	0.3660	0.7320
5	4.4902	0.7883	0.4011	0.8022
6	9.1811	1.0613	0.5364	1.0728
7	2.1214	1.4650	0.7834	1.5668

the origin of the discrepancy. For example, it appears that the Munich Method more closely satisfies the incorrect equation

$$p + (m/A)^2 v/2 = p_F$$

which contains the additional factor $\frac{1}{2}$ in the second term on the left. This can be seen by comparing columns 3 and 5 of Table 1. The differences between columns 3 and 5 might be attributable to computational inaccuracies in the computer program which implements the Munich Method. Regardless of the source of error, it is now clear that not only are the original combustion equations of the Munich Method incorrect, they are not even implemented correctly. This should not be too surprising given that the Munich Method also generates an isentropic process with a varying entropy.

The author of the Munich Method compares (Table 4.2–4.7)[8] calculated and experimental values of the mass flow rate and the vacuum specific impulse for several rocket engines. The comparison is between experimental data at a given expansion ratio, which he assembled from various sources, and two sets of calculated values. One set of calculated values came from the Munich Method with its finite-area combustion. The second set of calculated values apparently came from calculations which he carried out with an early version of the Lewis Code which did not include finite-area combustion. In this comparison, which would seem to favor the Munich Method over the Lewis Code, we find that the values from the old Lewis Code are always closer to his chosen experimental values than those from the Munich Method. Apparently this does not perturb him, it merely demonstrates the superiority of the Munich Method. His higher specific impulse values and mass flow rates simply demonstrate how much more improvement can still be achieved by improving the design of the engine's hardware components such as the turbopumps (p. 196).

Conclusions

This examination of the Munich Method for modeling rocket engine performance shows that it is not based on some new Alternative Theory, but is instead merely the application of standard thermodynamics. However, the implementation is beset by flaws in its thermodynamics, mathematics, and numerics. If the author corrected the errors then his calculations should produce values identical to those obtained from the Lewis Code. However, he would then still be burdened by an extremely inefficient algorithm for the calculation of nozzle expansions.

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